

# Prediction of Vibration Damping Properties of Polymer-Laminated Steel Sheet Using Time-Temperature Superposition Principle

FU-SEN LIAO\* and TZU-CHIEN J. HSU†

Institute of Materials Science and Engineering, National Sun Yat-Sen University, Kaohsiung 804, Taiwan, Republic of China

## SYNOPSIS

The vibration damping properties of the polymer-laminated steel sheet have been investigated theoretically and experimentally. The laminate consisted of a polymer layer, which was sandwiched between two steel sheets. Two polymers, a polyvinyl butyral and a copolymer of ethylene and acrylic acid, were used. The theoretical analysis was based on a model proposed by Ungar. A frequency analyzer was used to measure the loss factor of the laminate. The model required rheological data, such as storage modulus  $G'$  and loss tangent of the polymer at high frequency, which could not be obtained from commercially available dynamic rheometers. The time-temperature superposition principle was applied to the laminated polymer to construct the master curves of  $G'$  and loss tangent vs. frequency. These master curves provided rheological data at high frequency, which were superposed from data measured at low temperature. The results showed that a discrepancy existed between the loss factors predicted with superposed and without superposed data and the reasons for this discrepancy were discussed. The measured loss factors of the laminates at high frequency followed the predictions using superposed data, but not for those measured at low frequency. Factors accounting for this deviation were analyzed. The results also indicated that, in general, the transition temperature of the polymer-laminated steel sheet was 15°C–30°C higher than the corresponding glass transition temperature of the laminated polymer.

## INTRODUCTION

In recent years, there has been increasing interest in the development and utilization of polymer-laminated steel sheet for vibration damping purposes.<sup>1,2</sup> The laminate usually consists of a viscoelastic polymer layer, which is sandwiched between two steel sheets. The constrained polymer layer provides the laminate with satisfactory vibration and noise attenuation over a specific range of temperature. It is believed that the shear deformation of the constrained layer dominates the laminate damping and the loss tangent of the polymer used is of greatest importance in this type of damper.<sup>3–8</sup> Thus, noise is substantially reduced, mainly by the shear defor-

mation of the polymer laminated. Ungar et al.<sup>6–8</sup> proposed a theoretical approach to calculate the damping efficiency of a sandwich panel. Detailed derivations can be found in the literature. According to this approach, the loss factor  $\eta$  of the laminate used in this study can be represented by

$$\eta = \frac{\beta_2 Y X}{1 + (2 + Y)X + (1 + Y)(1 + \beta_2^2)X^2}$$

where  $Y$  is the stiffness parameter,  $X$  is the shear parameter of the laminate, and  $\beta_2$  is the shear loss tangent of the viscoelastic layer. In general, the stiffness parameter is a function of the elastic modulus, the thickness, and the density of the steel sheet. The shear parameter is a function of the elastic modulus and the thickness of the steel, the storage modulus and the thickness of the polymer, the applied frequency, and the density of the laminate (mass per unit area). The loss factor, defined above

\* Also at: New Materials R&D Department, China Steel Corporation, Hsiao Kang, Kaohsiung 802, Taiwan, R.O.C.

† To whom correspondence should be addressed.

as the ratio of the dissipated energy to the stored energy per vibration cycle, is a measure of vibration-damping efficiency. In practice, an iterative procedure is typically needed to calculate the value of  $\eta$ .<sup>9</sup>

Theoretical predictions of the damping properties of the polymer-based laminates have been investigated. Grates et al.<sup>10</sup> used a steel panel as the basic substrate with different constraining layers, such as aluminum, fiberglass filled epoxy, and epoxy laminates. The damping polymer used included polyvinyl acetate and some acrylate/acrylate interpenetrating polymer networks (IPNs). Using the percent critical damping method, the predicted damping curve provided a reasonably close approximation of the experimentally measured damping curves. However, the predictions were lower than experimental data by a factor of approximately 2. The reason was not explored. Chen et al.<sup>11</sup> applied Ungar's method to predict the loss factor of polymer-laminated steel sheet. The polymers used were a polyvinyl butyral and a copolymer of ethylene and acrylic acid. They found that the theory predicted a higher loss factor than the experimental data. According to their analysis, the errors might come from, among other sources, primarily the extrapolation of the rheological data of polymer from low to high frequency.

Generally speaking, the acoustical frequency of practical interest is in the range of 20 Hz to 20,000 Hz. To apply Ungar's model to the laminated steel sheet successfully, one has to obtain the storage modulus  $G'$  and the loss tangent of a viscoelastic polymer from the dynamic rheometer at the required frequency range. However, measurement of these viscoelastic properties on any commercially available instrument is limited to a narrow frequency range, that is, only up to 100 Hz or so. Consequently, the theoretical approach has been restricted by applying only low frequency data of polymers in the predictions.<sup>10,11</sup> This results in an inherent error. However, with the aid of the time-temperature superposition principle,<sup>4,12</sup> this error can be greatly reduced.

It has been well established that in a viscoelastic polymer, an increase in frequency gives an equivalent decrease in temperature and vice versa. There exists an empirical correspondence between frequency and temperature. To obtain rheological data at high frequencies, one needs to conduct the dynamic rheological measurements at low temperatures. Based on the time-temperature superposition principle, the effective frequency range of the viscoelastic properties can be extended. This involves measuring the viscoelastic properties (storage modulus and loss tangent of the viscoelastic layer) over a limited frequency range and at various tempera-

tures. Plots of rheological properties vs. frequency are constructed and shifted along the frequency axis about the reference temperature until a single smooth and continuous curve results. The constructed curve is known as the "master curve."

In this study, two polymers were selected, one for the room-temperature type and the other for the high-temperature type damper. Temperature and frequency dependencies of the loss factor of the laminated steel sheet, based on these polymers, are investigated experimentally. The time-temperature superposition principle is applied for the polymers and the superposed data are adopted in Ungar's theoretical approach to predict the loss factors of the laminates. Comparisons between the experimental data and the theoretical predictions are also addressed.

## EXPERIMENTAL

### Dynamic Rheological Measurements

Two types of polymer layers were used in this study, a polyvinyl butyral (PVB B-08M, Chang Chun Petrochemical Co.) and a copolymer of ethylene and acrylic acid (PEAA, Primacor 3440, Dow Chemical Co.). The storage modulus and the loss tangent of these two polymers were examined with a dynamic rheometer (RDS-II, Rheometrics). The frequency used ranged from 0.1 Hz to 60 Hz. The rectangular testing specimens had a dimension of  $42.0 \times 12.0 \times 3.0$  mm. In order to ensure that the dynamic mechanical tests were all in the linear viscoelastic region of the polymer, several strain-sweep experiments were performed before measurements. The shear strains in the determinations of  $G'$  and loss tangent were then set at 0.05% for PVB and at 0.02% for PEAA.

### Frequency Spectrum Analysis

The laminated steel sheet was composed of two 0.50-mm-thick steel sheets, laminated with a polymer film of 0.12 mm thickness. The raw steel sheet used was cold-roll steel (SPCC, China Steel Corp.). Specimen preparation and vibration-damping measurement in this study were conducted following the standard method ASTM E 756-83. To provide the spacer material in the root section of the specimens, as required in ASTM, a 0.12-mm-thick aluminum plate was used to replace the polymer film in the root section. Lamination was done in a thermal press at 180°C. The laminate was then cut into specimens

10 mm in width and 205 mm in length, with a free section of 180 mm and a root section of 25 mm in length. The root section of the specimens was adhesively bonded with another, 0.50-mm thick steel piece on either side of the laminated panel and then was closely clamped in a rigid fixture.

Shown in Figure 1 is the schematic diagram illustrating the experimental set-up for the vibration damping test. Before vibration testing, the PVB-based and PEAA-based laminates were aged for 3 h at 100°C and 70°C, respectively. During testing, random noise excitation was sent to the specimen via a magnetic transducer (Type MM 0002, B & K Co., Denmark) to force the laminate to vibrate. The response was detected by a capacitive transducer (Type MM 0004, B & K Co.). Both input and output signals were fed into a fast Fourier transform signal analyzer (Type 2032, B & K Co.) and then were analyzed in the frequency range of 0 to 1600 Hz with the Hanning window function. The frequency spectra were continuously recorded during the test. Zoom analysis was also performed to improve the resolution and the accuracy of the spectrum for each resonance peak. The loss factor was then calculated around each resonance peak using the half-power bandwidth method. A theoretical description of the damping efficiency by this method was mentioned elsewhere.<sup>9,13</sup> In general, there were four resonance frequency peaks observed, that is, four resonance

modes (starting from mode 1 for the lowest resonance frequency to mode 4 for the highest frequency).

## RESULTS AND DISCUSSION

### Time-Temperature Superposition Principle

Figure 2 shows the temperature dependencies of loss tangent for PVB and PEAA, measured at 1 Hz. PVB shows a glass transition temperature of around 70°C and a peak value of loss tangent of 1.73, whereas PEAA experiences a glass transition around 30°C with a relatively low peak value of loss tangent of 0.26. Since the vibration damping of polymer-laminated steel sheet is dominated by the shear deformation of the polymer laminated, it is rationalized that the optimum temperature at which the maximum loss factor occurs for the laminate should be around the glass transition temperature of the polymer.

A series of measurements on frequency dependence of dynamic mechanical properties were performed at various temperatures near the corresponding glass transition temperatures for both PVB and PEAA (Figs. 3-6). The testing frequencies for these measurements were in the range of 0.1 Hz to 60 Hz, due to the mechanical limitation of the in-

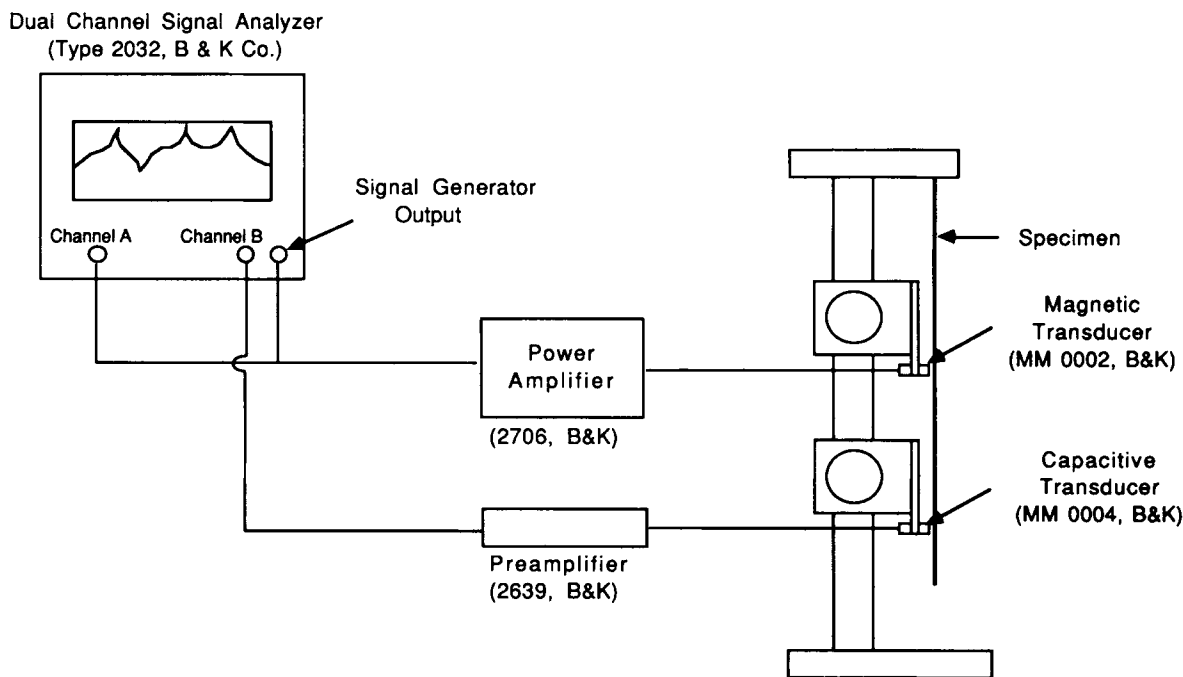
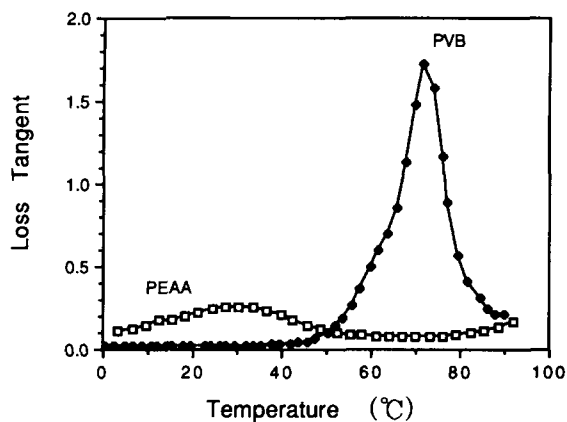
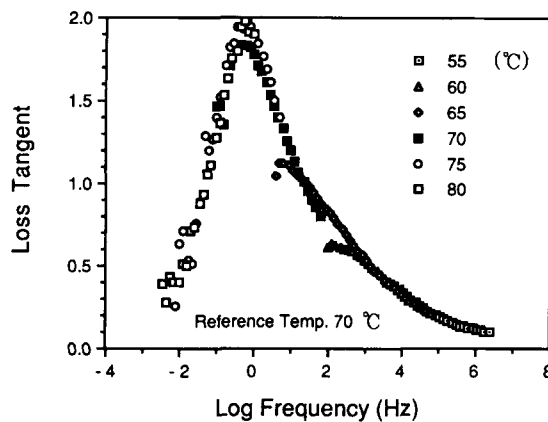


Figure 1 Schematic diagram illustrating the vibration damping test.



**Figure 2** Temperature dependence of loss tangent for PVB and PEAA at 1 Hz.



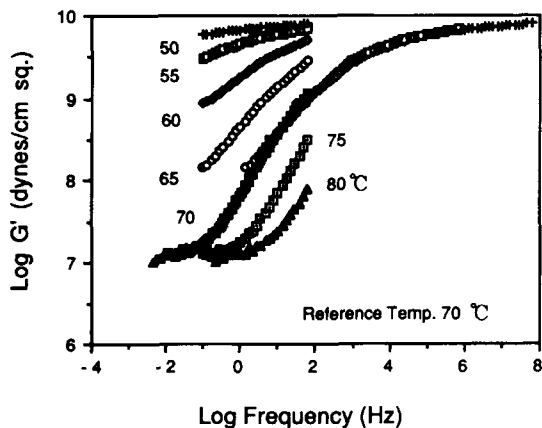
**Figure 4** Frequency dependence of loss tangent measured at various temperatures and the superposed spectrum with reference temperature 70°C for PVB.

strument and the requirement of linear viscoelasticity in the polymer used. However, the vibration frequency of practical interest is much higher than 60 Hz. Following the time-temperature superposition principle, a dynamic mechanical spectrum at a constant temperature can be extended to a frequency range beyond the instrument-limited region by shifting other spectra, which are obtained at different temperatures. Figure 3 shows plots of the measured shear storage modulus ( $G'$ ) vs. frequency of PVB at various temperatures. Also shown in this figure is the master curve, which is constructed by horizontally shifting each curve with respect to the reference temperature 70°C.

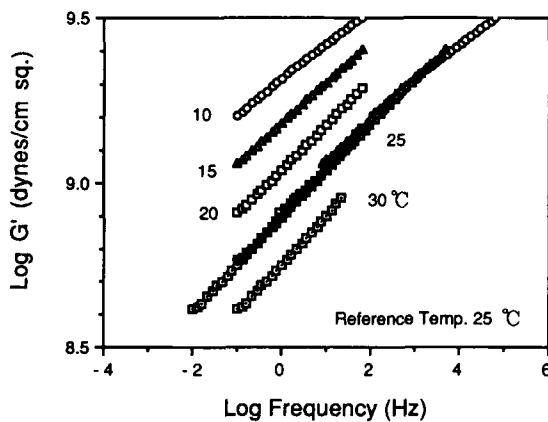
In Figure 3, the low temperature data have been shifted to the right to become the high frequency data, and vice versa. The same procedure is applied to obtain the master curve of loss tangent vs. fre-

quency for PVB, as demonstrated in Figure 4. Similar treatments are applied to PEAA and the results are shown in Figures 5 and 6 with the reference temperature 25°C. The master curves so constructed in Figures 3-6 seem reasonably smooth. The frequency range has been extended to as high as  $10^8$  Hz. This justifies the use of these superposition data in the subsequent predictions of the loss factors of the laminates.

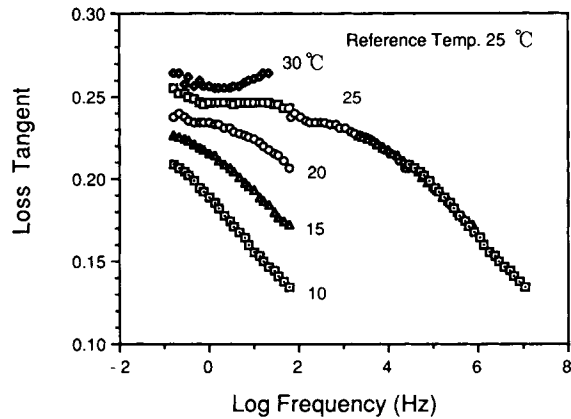
Figure 3 shows that  $\log G'$  of PVB covers 3 orders of magnitude, that is, from 7-10. On the other hand,  $\log G'$  data of PEAA in Figure 5 covers less than 1 order of magnitude, that is, from 8.5-9.5. It is apparent from Figures 3 and 5 that  $\log G'$  increases with increasing frequency. This suggests that high frequency makes the polymer more stiff, thus resulting in a high  $G'$ . As for loss tangent data, it



**Figure 3** Frequency dependence of  $G'$  measured at various temperatures and the superposed spectrum with reference temperature 70°C for PVB.



**Figure 5** Frequency dependence of  $G'$  measured at various temperatures and the superposed spectrum with reference temperature 25°C for PEAA.



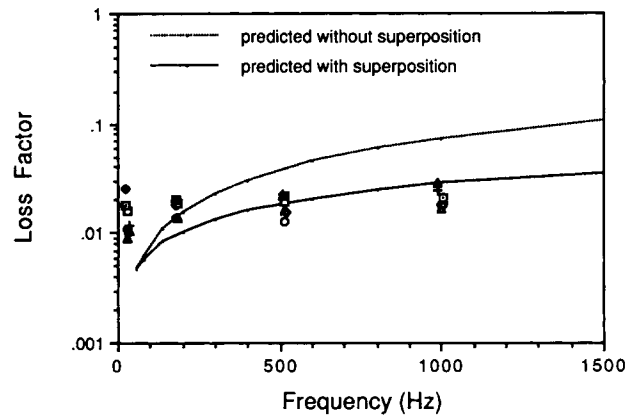
**Figure 6** Frequency dependence of loss tangent measured at various temperatures and the superposed spectrum with reference temperature 25°C for PEA.

ranges from 0.2–2.0 for PVB and only 0.13–0.27 for PEA, as illustrated in Figures 4 and 6. In Figure 4, PVB shows a maximum loss tangent of *ca.* 2.0 at 1 Hz. Above 1 Hz, the loss tangent decreases with increasing frequency. Below 1 Hz, the loss tangent increases with increasing frequency. On the other hand, PEA in Figure 6 shows no maximum, and loss tangent decreases with increasing frequency. In other words, frequency dependence of  $G'$  and loss tangent for PVB is much stronger than that of PEA. The significance of this difference becomes obvious when comparisons of theoretical predictions with and without superposition data are made in the next section.

### Theoretical Predictions

In order to compare the theoretically predicted damping efficiencies of the laminates with the experimental data, a computer program was written to calculate the loss factors ( $\eta_s$ ) of the laminates. The program incorporated parameters affecting the damping efficiency, which included the thickness and density of both steel and polymer, the elastic modulus of the steel, the frequency, the temperature, and the storage modulus ( $G'$ ) and loss tangent of the viscoelastic layer. With this program, it was possible to predict the effect of temperature and frequency on the laminate damping.

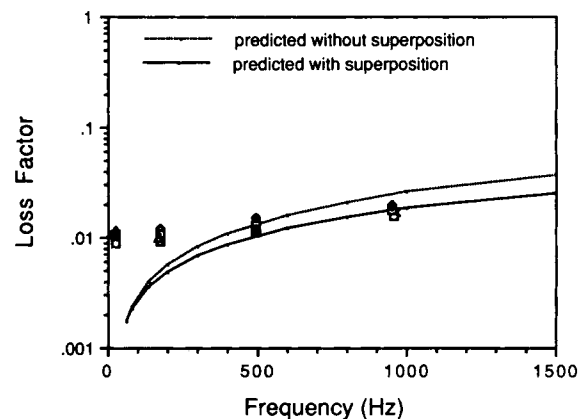
The results for PVB-based laminates at 70°C and PEA-based laminates at 25°C are shown in Figures 7 and 8, respectively. The solid lines in Figures 7 and 8 demonstrate the predicted  $\eta_s$  of the laminates, using the superposition data adopted from the master curves shown in Figures 3 to 6. To illustrate the



**Figure 7** Loss factors measured and predicted vs. frequency for PVB-based laminate at 70°C. Different symbols are repeated experimental data.

effect of the time-temperature superposition principle, data of  $G'$  and loss tangent measured at 60 Hz, without superposition, were also directly applied in the predictions (dashed lines in Figs. 7 and 8).

It is evident from Figures 7 and 8 that, except at 60 Hz, the applications of loss tangent and  $G'$  without superposition in the model lead to higher predicted values than those employing superposed data. At 60 Hz,  $\eta_s$  from both predictions are the same, since no superposition is applied. The discrepancy becomes larger when the frequency gets higher. This discrepancy can be rationalized when one compares loss tangent data in Figures 4 and 6, since the shear loss tangent dominates primarily the laminate damping. In both figures, loss tangent at frequencies higher than 60 Hz are always lower than that at 60 Hz.



**Figure 8** Loss factors measured and predicted vs. frequency for PEA-based laminate at 25°C. Different symbols are repeated experimental data.

The discrepancy can further be understood if one follows the analysis of Ungar's approach. Detailed derivations can be found in the literature.<sup>9</sup> According to this approach, there exists an optimum shear parameter  $X_0$  for a given laminate configuration, that is, at fixed  $\beta_2$  and  $Y$ . If  $X$  is greater than  $X_0$ ,  $\eta$  varies as  $1/X$ ; for  $X$  less than  $X_0$ ,  $\eta$  varies as  $X$ . Shown in Table I are the calculated  $X$  and  $X_0$  used in the predictions of Figures 7 and 8. For both PVB-based and PEAA-based laminates, Table I indicates that  $X$ s are always higher than  $X_0$  in all predictions. Thus,  $\eta$  should vary as  $1/X$  within the applied frequency range of 60 Hz–1500 Hz. Furthermore,  $X$ s in the predictions with superposed data are higher than  $X$ s in the predictions without superposition. Accordingly, the predicted  $\eta$ s with superposed data are lower than those without superposition.

In addition, since  $\eta$  varies as  $1/X$  and  $X$  varies as  $G'/\omega_n$ , where  $\omega_n$  is the  $n$ th mode of the resonant frequency of the laminate,<sup>7</sup>  $\eta$  should decrease with increasing  $G'$  at fixed  $\omega_n$ . Figures 3 and 5 indicate that the superposed  $G'$  is greater than  $G'$  without superposition for frequencies above 60 Hz. This increase in  $G'$  also contributes to the depression of predicted  $\eta$ s with superposition data.

Comparing Figures 7 and 8, the discrepancy for PVB-based laminates is larger than the discrepancy for PEAA-based laminates. This is primarily due to the strong frequency dependencies of  $G'$  and loss tangent for PVB, as mentioned in the previous section. For those less frequency-sensitive polymers, no substantial difference exists between the theoretical predictions with superposition and without superposition data.

### Comparisons between Theoretical and Experimental Data

Figures 7 and 8 also indicate that the measured  $\eta$ s at high frequencies (mode 3 and mode 4) follow closely those predicted using superposed data, but the experimental results at low frequencies (mode 1 and mode 2) are generally higher than the predicted results. Different symbols in these figures indicate repeated experimental data. Several factors may contribute to this deviation at low frequencies. The model does not take into account: first, the adhesive bonding at the interface between the polymer and the steel sheet and, second, the flatness of the steel sheet, which makes it difficult to provide a laminate with uniform thickness of polymer interlayer. The above-mentioned phenomena may not exist at high frequencies, but may promote laminate damping at low frequencies. Also, the theory was developed using sinusoidal expansion for the mode shape of vibration. This approximation was acceptable only at high frequencies. At low frequencies, the edge effect of the vibrating laminate becomes significant, leading to a large misfit between the observed and the calculated values. Furthermore, the force level for these low frequency tests may be beyond the limit of linear viscoelasticity of the polymers, and the depression of  $G'$  in the nonlinear region leads to higher damping. The results indicate that theoretical predictions using superposed data are feasible at high frequencies.

The comparisons also reveal that the  $\eta$ s of PVB-based laminates at 70°C are approximately the same as those of PEAA-based laminates at 25°C. The

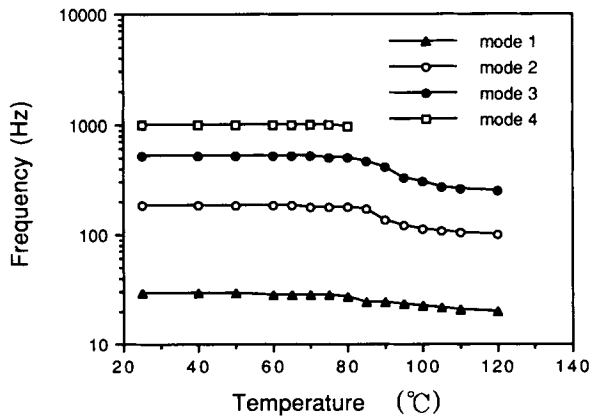
**Table I** Calculated Shear Parameter  $X$  and Optimum Shear Parameter  $X_0$  in the Predictions of Figures 7 and 8

Frequency (Hz)	PVB-Based Laminate				PEAA-Based Laminate			
	Without Superposition		With Superposition		Without Superposition		With Superposition	
	$X$	$X_0$	$X$	$X_0$	$X$	$X_0$	$X$	$X_0$
60	85.18	0.33	85.18	0.33	109.12	0.41	109.12	0.41
80	63.78	0.33	69.92	0.33	81.74	0.41	85.65	0.41
140	36.27	0.33	46.80	0.34	46.53	0.41	51.64	0.41
200	25.27	0.33	36.65	0.35	32.45	0.41	37.59	0.41
300	16.71	0.33	27.28	0.35	21.50	0.41	26.12	0.41
400	12.44	0.33	22.37	0.36	16.03	0.41	20.16	0.41
600	8.17	0.33	16.64	0.36	10.56	0.41	13.91	0.41
800	6.03	0.33	13.61	0.37	7.82	0.41	10.67	0.41
1000	4.75	0.33	11.35	0.37	6.18	0.41	8.68	0.41
1500	3.06	0.33	8.51	0.38	4.00	0.41	5.93	0.41

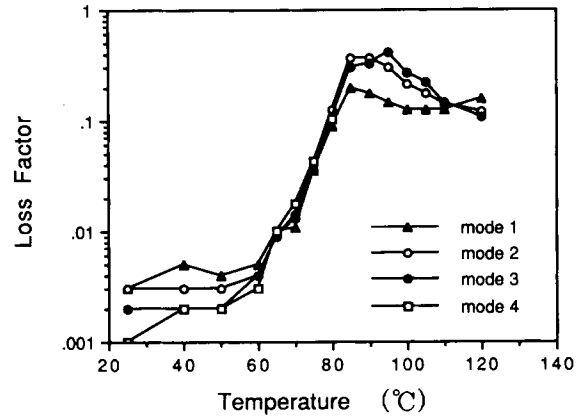
corresponding values of loss tangent for maximum damping (Figs. 3 and 5) are 1.73 for PVB and 0.26 for PEAA at 1 Hz. Apparently, a difference of peak damping temperature exists between the laminate and the polymer. A difference in frequency applied certainly accounts for the different damping behavior between the laminate and the polymer. In addition, since the polymer is constrained between two steel sheets, the sandwich-type structure makes the laminated polymer more stiff and makes it more difficult for shear deformation to occur effectively. Thus, it is rationalized that maximum laminate damping should take place at a temperature higher than the corresponding glass transition temperature of the polymer. Since the reference temperatures used in Figures 3-6 of PVB and PEAA are all below the transition temperatures of the laminates, no obvious difference of laminate damping could be observed in Figures 7 and 8 accordingly.

**Frequency Spectrum Analysis**

To explore further the different transition behavior mentioned above, a series of damping measurements of the laminates was conducted. Figures 9 and 10 are the results for the PVB-based laminates at various temperatures. Figure 9 shows the temperature dependence of resonance frequency of each mode. In theory, the resonance frequency of a single degree-of-freedom damping system is proportional to the square root of Young's modulus of the system concerned for each mode.<sup>13</sup> Therefore, the logarithmic plot of resonance frequency vs. temperature in Figure 9 can be interpreted as temperature dependence of the storage modulus of the constrained polymer layer or, in effect, the apparent storage modulus of the laminate. It is observed from Figure 9 that the



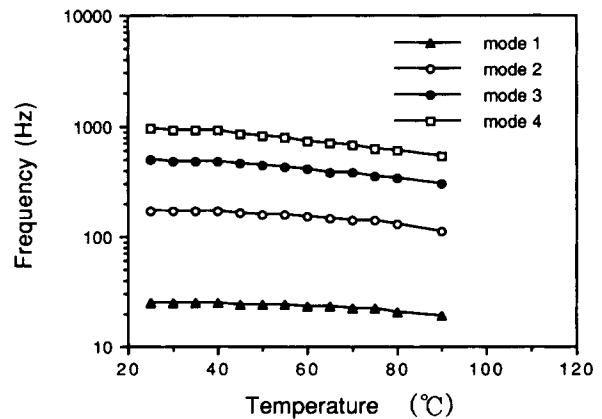
**Figure 9** Plots of resonance frequency vs. temperature for PVB-based laminate.



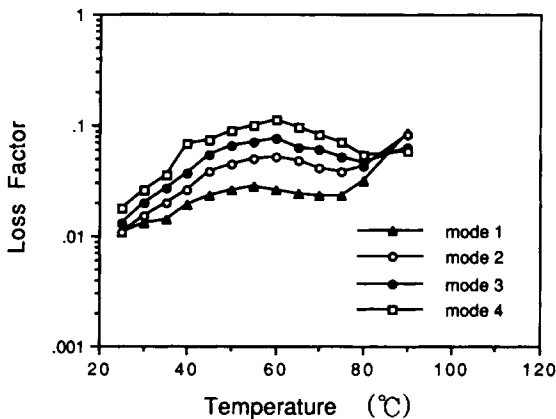
**Figure 10** Temperature dependence of loss factor for PVB-based laminate.

resonance frequency of each mode shows a transition as the testing temperature increases. Plots of  $\eta_s$  vs. temperature of the PVB-based laminates are shown in Figure 10. The  $\eta_s$  in this figure appear to be no more than 0.02 since the testing temperatures are below 70°C. Above this temperature,  $\eta_s$  remarkably increase. The transition temperature at which maximum ( $\eta_{max}$ ) occurs is in the range of 85°C to 95°C for those observable modes. However, the glass transition temperature of PVB was observed to be 70°C at 1 Hz and should be higher than 70°C at higher frequencies. Thus, the effective temperature of the PVB-based laminate for maximum damping is generally 15°C to 25°C higher than the glass transition temperature of PVB.

Figure 11 illustrates the temperature dependence of resonance frequency for the PEAA-based laminates. It is seen that the dependencies for all modes are less clear and no obvious transition can be observed. As illustrated in Figure 12, the measured  $\eta_s$



**Figure 11** Plots of resonance frequency vs. temperature for PEAA-based laminate.



**Figure 12** Temperature dependence of loss factor for PEAA-based laminate.

of the PEAA-based laminates appear to be around 0.015 since the testing temperature is at 25°C. The transition temperature of the laminate falls between 55°C–60°C, which is generally 25°C–30°C higher than the corresponding transition temperature of PEAA.

Both Figures 10 and 12 show that for both PVB-based and PEAA-based laminates, the transition temperature increases with increasing mode order;  $\eta$  also increases with increasing mode order.  $\eta_{\max}$  of the PVB-based laminate ranges from 0.20 to 0.41 (Fig. 10), which is generally higher than that of PEAA-based laminate ( $\eta_{\max}$  from 0.03–0.11 in Fig. 12). This is consistent with the results observed in Figure 1. It can be stated, in general, that the transition temperature of the polymer-laminated steel sheet is 15°C–30°C higher than the corresponding transition temperature of the laminated polymer.

## CONCLUSIONS

The vibration damping properties of the polymer-laminated steel sheet have been investigated theoretically using Ungar's model. The model required rheological data such as storage modulus  $G'$  and loss tangent of the polymer at high frequency, which could not be obtained from commercially available dynamic rheometers. The time-temperature superposition principle was applied to the laminated polymer to construct the master curves of  $G'$  and loss tangent vs. frequency. These master curves provided rheological data at high frequency, which

were superposed from data measured at low temperature.

The predicted loss factors of the laminates using superposed data were lower than those predicted without superposition. Comparisons between the predictions using superposed rheological data and the experimental data showed that the loss factors measured at high frequency obeyed the predictions, but those measured at low frequency were generally higher than predicted. Factors, such as edge effect and nonlinear viscoelastic behavior of the constrained polymers, may contribute to this deviation. The results indicated that, in general, the transition temperature of the polymer-laminated steel sheet was 15°C–30°C higher than the corresponding glass transition temperature of the laminated polymer.

The authors wish to thank Professor A. C. Su at IMSE for constructive guidance in performing the time-temperature superposition principle and Dr. S. I. Chen at CSC for helpful discussion. Also appreciated is Mr. M. S. Chang at CSC for the preparation of the laminates.

## REFERENCES

1. A. Jouet and M. Glemet, In *Hot and Cold-Rolled Sheet Steels*, R. Pradhan and G. Ludkovsky, Eds., TMS, Warrendale, PA, 1988, p. 221.
2. N. Chiba, *Nippon Kokan Technical Report*, Fukuyama, Japan, **43**, 50 (1985).
3. G. L. Ball and I. O. Salyer, *J. Acoust. Soc. Am.*, **39**, 663 (1966).
4. D. I. G. Jones, *J. Sound and Vibration*, **33**, 451 (1974).
5. H. Mizumachi, *J. Adhesion*, **2**, 292 (1970).
6. E. E. Ungar, *J. Acoust. Soc. Am.*, **34**, 1082 (1962).
7. D. Ross, E. E. Ungar, and E. M. Kerwin, Jr., In *Structural Damping*, J. E. Ruzicka, Ed., ASME, New York, 1959, Sec. 3.
8. E. M. Kerwin, Jr., *J. Acoust. Soc. Am.*, **31**, 952 (1959).
9. E. E. Ungar, In *Noise and Vibration Control*, L. L. Beranek, Ed., McGraw-Hill, New York, 1979, Chap. 14.
10. J. A. Grates, D. A. Thomas, E. C. Hickey, and L. H. Sperling, *J. Appl. Polym. Sci.*, **19**, 1731 (1975).
11. Y. S. Chen, T. J. Hsu, and S. I. Chen, *Metall. Trans.*, **22A**, 653 (1991).
12. J. D. Ferry, *Viscoelastic Properties of Polymers*, 3rd Ed., Wiley, New York, 1980.
13. A. D. Nashif, D. I. G. Jones, and J. P. Henderson, *Vibration Damping*, Wiley, New York, 1985.

Received May 7, 1991

Accepted August 28, 1991